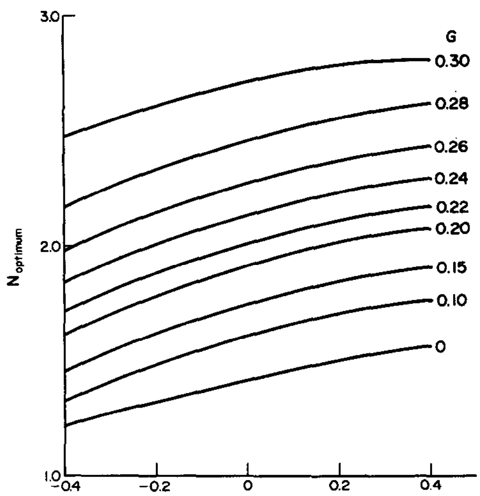

FIG. 2. Optimisation chart for rectangular fin with uniform internal heat generation and temperature dependent thermal conductivity.

The optimisation data is presented in Fig. 2 wherein N_{opt} obtained by solving equation (9) is plotted against ε for a range of values of G. With the aid of Fig. 2, the optimum dimensioned fin for a specified heat generation can be readily designed allowing for thermal conductivity variation.

REFERENCES

1. W. S. Minkler and W. T. Rouleau, The effects of internal heat generation on heat transfer in thin fins, *Nucl. Sci. Engng* 7, 400–406 (1960).
2. G. B. Melese and J. E. Wilkins, Efficiency of longitudinal fins of arbitrary shape with nonuniform surface heat transfer and internal heat generation, in *Proceedings Third International Heat Transfer Conference*, Vol. III, pp. 272–280. A.I.Ch.E., New York (1966).
3. G. Ahmadi and A. Razani, Some optimisation problems related to cooling fins, *Int. J. Heat Mass Transfer* **16**, 2369–2375 (1973).
4. A. Aziz and S. M. Enamul Huq, Perturbation solution for convecting fin with variable thermal conductivity, *J. Heat Transfer* **97C**, 300–301 (1975).
5. R. J. Krane, Discussion on a previously published paper by A. Aziz and S. M. Enamul Huq, *J. Heat Transfer*. To be published.
6. D. Q. Kern and A. D. Kraus, *Extended Surface Heat Transfer*, pp. 193–199. McGraw-Hill, New York (1972).

Int. J. Heat Mass Transfer. Vol. 20, pp. 1255–1257. Pergamon Press 1977. Printed in Great Britain

INVESTIGATION OF TEMPERATURE FIELDS IN HEAT EXCHANGERS OF POROUS CYLINDRICAL BOARDS

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NOMENCLATURE

A, B, dimensionless parameters;
C_{12}, C_{13}, constants;
C_p, specific heat of the fluid;
J_0, J_1, Bessel functions;
K_i, coefficients of evolution of temperature field;
$\dot{Q}$, total heat flow;
$\dot{Q}_0$, ideal heat flow;
R, $= r/r_0$, dimensionless radius, co-ordinate;
r, radius, co-ordinate;
r_0, radius of porous board;
T_f, temperature of fluid;
T_{f0}, temperature of incoming fluid;
T_s, temperature of porous material;
T_{s0}, temperature of circumference of porous board;
w, specific mass throughflow;
Z, $= z/r_0$, dimensionless co-ordinate;
z, co-ordinate;
Z_0, $= z_0/r_0$, dimensionless height of porous board;
z_0, height of porous board.

Greek symbols
α, coefficient of heat transfer;
α_i, zero points of function J_0;
β, specific area of heat transfer;
γ, $= \lambda_z/\lambda_r$, rate of orthotropy;
Θ_s, dimensionless temperature of porous material;
Θ_f, dimensionless temperature of fluid;
λ_{ii}, roots of characteristic polynome;
λ_z, λ_r, thermal conductivities in axial and in radial directions;
ϕ_i, component of temperature field;
φ, co-ordinate.

INTRODUCTION

THE HEAT exchangers of porous materials are of importance in number of applications, for example in an effective utilization of enthalpy of outgoing gaseous helium in throughflow cryostats, in refrigerators making use of dissolution of ^3He in ^4He [1].

This article presents a solution of stabilized temperature fields in a orthotropic porous material of cylindrical shape, thermally connected by its circumference to a body with temperature T_{s0}. It can be higher or lower than the temperature of incoming cooling (warming) medium T_{f0}, flowing only in axial direction through the porous material (Fig. 1).

The solution is found on the following assumptions:

(a) The geometrical and physical parameters are axially symmetrical.

(b) We regard the porous substance as a continuous and homogeneous environment (i.e. neglecting the microstructure).

(c) The heat is brought in solely by the outer circumference of the porous board, and removed by a transfer into the fluid, or conversely.

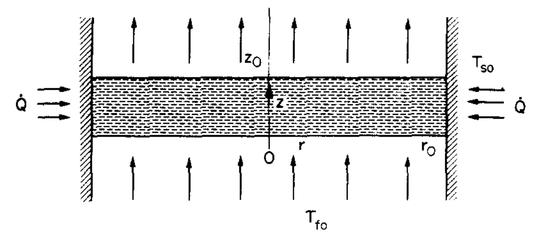

FIG. 1. A schematical section through the exchanger being studied, with a denotation of the co-ordinates r, z and the direction of the flow of the fluid and of the heat.

(d) The fluid thermal conductivity is negligible in comparison to the solid thermal conductivities λ_r and λ_z.

(e) Within the respective temperature intervals, thermal conductivity, the heat-transfer coefficient and the specific heat are regarded as constants [2]. When the coefficients change considerably, the presented case can be used as a first approximation of the iterative procedure.

DEDUCTION OF ELEMENTARY EQUATIONS

The solution is carried out in cylindrical co-ordinates. In view of condition (a) the temperature fields are independent of co-ordinate φ. In the verbal description we shall assume that $T_{s0} > T_{f0}$, i.e. that flowing fluid draws heat from the porous substance. In opposite case the same relations hold of course good.

We can derive elementary equations if we compare heat conduction in a porous substance caused by a temperature gradient, heat transfer to the fluid and heating of flowing fluid [3]. We express them in dimensionless form. We define dimensionless temperatures by equations

$$\Theta_s(R,Z) = \frac{T_s(R,Z) - T_{f0}}{T_{s0} - T_{f0}}, \quad (1)$$

$$\Theta_f(R,Z) = \frac{T_f(R,Z) - T_{f0}}{T_{s0} - T_{f0}}. \quad (2)$$

The elementary equations have the dimensionless form

$$\left[\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} + \gamma\frac{\partial^2}{\partial Z^2}\right]\Theta_s(R,Z) = A\frac{\partial \Theta_f(R,Z)}{\partial Z}, \quad (3)$$

$$\frac{\partial \Theta_f(R,Z)}{\partial Z} = B[\Theta_s(R,Z) - \Theta_f(R,Z)], \quad (4)$$

where

$$A = \frac{wC_p r_0}{\lambda_r}, \quad B = \frac{\alpha\beta r_0}{wC_p}, \quad \gamma = \frac{\lambda_z}{\lambda_r} \quad (5)$$

are dimensionless parameters.

The boundary conditions appertaining to equations (3) and (4) are

$$\Theta_s(1,Z) = 1, \quad (6)$$

$$\frac{\partial \Theta_s(R,0)}{\partial Z} = \frac{\partial \Theta_s(R,Z_0)}{\partial Z} = 0, \quad (7)$$

$$\Theta_f(R,0) = 0, \quad (8)$$

where (6) reflects the constant temperature of the circumference of the porous board, (7) secures the zero flux of heat across the upper and lower boundaries of the porous substance, and (8) determines the temperature of the incoming fluid.

ANALYTICAL SOLUTION

From equations (3) and (4) a homogeneous equation for a single temperature function can be derived:

$$\left[\frac{\partial^3}{\partial R^2\,\partial Z} + \gamma\frac{\partial^3}{\partial Z^3} + \frac{1}{R}\frac{\partial^2}{\partial R\,\partial Z} + B\frac{\partial^2}{\partial R^2} + \gamma B\frac{\partial^2}{\partial Z^2} + \frac{B}{R}\frac{\partial}{\partial R} - AB\frac{\partial}{\partial Z}\right]\Theta_s(R,Z) = 0. \quad (9)$$

By a solution (9) via a separation of the variables we can obtain [3] the expression of the temperature field in a porous substance

$$\Theta_s(R,Z) = 1 - \sum_{l=1}^{\infty} K_l J_0(\alpha_l R) \times [e^{\lambda_{l1}Z} + C_{l2}\,e^{\lambda_{l2}Z} + C_{l3}\,e^{\lambda_{l3}Z}]. \quad (10)$$

From the boundary conditions we can determine the values of the constants which appear in relationship (10). If condition (6) is to be complied with, all the terms of the sum must be zero for $R = 1$, i.e. α_l are the zero points of the Bessel function of the zero order

$$J_0(\alpha_l) = 0. \quad (11)$$

We consider only the positive values α_l. The constants λ_{li} are the roots of a characteristic polynome which was produced by the solution of equation (9) by separation of the variables:

$$\gamma\lambda_l^3 + \gamma B\lambda_l^2 - (\alpha_l^2 + AB)\lambda_l - \alpha_l^2 B = 0. \quad (12)$$

If we require fulfilment of (7) in respect of each term of the sum in (10), this condition will lead to an algebraic system of equations for constants C_{l2} and C_{l3}

$$\begin{bmatrix} \lambda_{l2}; & \lambda_{l3} \\ \lambda_{l2}\,e^{\lambda_{l2}Z_0}; & \lambda_{l3}\,e^{\lambda_{l3}Z_0} \end{bmatrix}\begin{bmatrix} C_{l2} \\ C_{l3} \end{bmatrix} = \begin{bmatrix} -\lambda_{l1} \\ -\lambda_{l1}\,e^{\lambda_{l1}Z_0} \end{bmatrix}. \quad (13)$$

The last unknown constants shall be determined from condition (8), which has however been formulated for a different temperature function.

From (3) and (4) it follows that

$$\left[\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} + \gamma\frac{\partial^2}{\partial Z^2} - AB\right]\Theta_s(R,Z) = -AB\Theta_f(R,Z), \quad (14)$$

hence according to (8) for $Z = 0$ the following applies

$$\left[\frac{\partial^2}{\partial R^2} + \frac{1}{R}\frac{\partial}{\partial R} + \gamma\frac{\partial^2}{\partial Z^2} - AB\right]\Theta_s(R,0) = 0. \quad (15)$$

Equation (15) is a boundary condition equivalent to (8), but formulated for function Θ_s.

After substituting (10) to (15) and using completeness and orthogonality of the Bessel functions $J_0(\alpha_l R)$ we can derive expression [3, 4]

$$K_l = \frac{2AB}{\alpha_l J_1(\alpha_l)}[\phi_l(0)]^{-1}, \quad (16)$$

where

$$\phi_l(Z) = (\alpha_l^2 - \gamma\lambda_{l1}^2 + AB)\,e^{\lambda_{l1}Z} + C_{l2}(\alpha_l^2 - \gamma\lambda_{l2}^2 + AB)\,e^{\lambda_{l2}Z} + C_{l3}(\alpha_l^2 - \gamma\lambda_{l3}^2 + AB)\,e^{\lambda_{l3}Z}. \quad (17)$$

The temperature field in a porous substance is fully determined now.

By substituting (10) into (14) we shall obtain the analytical expression of temperature field in the fluid passing through

$$\Theta_f(R,Z) = 1 - \frac{1}{AB}\sum_{l=1}^{\infty} K_l\phi_l(Z)J_0(\alpha_l R). \quad (18)$$

From the temperature field in the porous substance (10) we can also express a total flow of the heat drawn by the exchanger

$$\dot{Q} = 2\pi r_0\lambda_r(T_{s0} - T_{f0})\sum_{l=1}^{\infty}\alpha_l K_l J_1(\alpha_l) \times \int_0^{Z_0}[e^{\lambda_{l1}Z} + C_{l2}\,e^{\lambda_{l2}Z} + C_{l3}\,e^{\lambda_{l3}Z}]\,dZ. \quad (19)$$

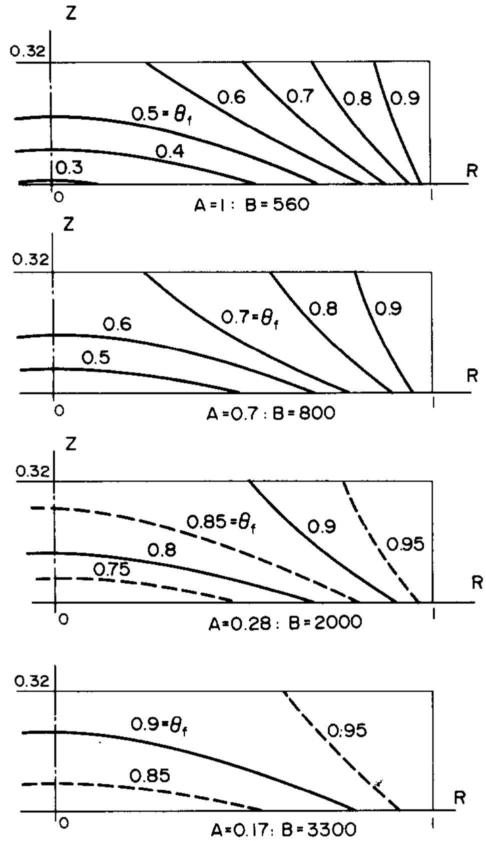

FIG. 2. Temperature fields in the fluid at different through-flows expressed by means of dimensionless isotherms Θ_f = const. The situation corresponds to $r_0 = 25$ mm, $z_0 = 8$ mm, $\gamma = 0.2$, $AB = \alpha\beta r_0^2/\lambda_r = 560$ = const.

This flow can be compared with an ideal off-take of heat when the fluid that passed through will reach the temperature T_{s0}. To this corresponds the heat flux drawn off

$$\dot{Q}_0 = A\lambda_r \pi r_0 (T_{s0} - T_{f0}). \quad (20)$$

By comparing the heat flows $\dot{Q}$ and $\dot{Q}_0$ we can judge exactly in how far the fooling capacity of the fluid passing through has been exploited.

NUMERICAL SOLUTION

The above relations for the temperature fields have been calculated on an HP 9830A calculator, using the first twelve terms with a correction for the rest of the series.

If $B \gg 1$ the temperature of fluid is practically equal to the temperature of the porous material in the whole volume except the layer at the plane $Z = 0$ with thickness of the order B^{-1}. If we change α and β ($B \gg 1$ and other parameters are constants) the new temperature fields are practically the same except thin layer at $Z = 0$. If $B \gg 1$ only refining of porous material cannot improve the quality of exchanger.

Figure 2 indicates the calculated temperature fields by means of isotherms in the fluid with different flows. From the diagrams it is clear that with the throughflow increasing (i.e. the A growing), the temperature of the central portion decreases and the isotherms become denser. The change of B in the interval $B > 30$ does not move practically the isotherms on this pictures.

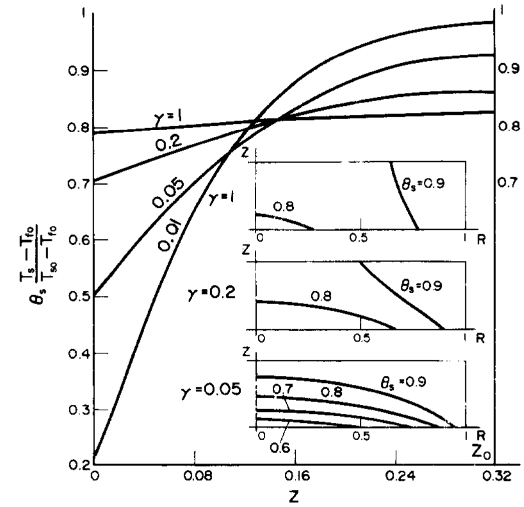

FIG. 3. Dependences of dimensionless temperature Θ_s of the porous board at its centre ($R = 0$) on the co-ordinate Z for different anisotropies of the porous material $\gamma = \lambda_z/\lambda_r$. Temperature of fluid $\Theta_f \doteq \Theta_s$. The dependences correspond to $A = 0.28$, $B = 2000$, $\lambda_r = 25$ W/mK.

In Fig. 3 there are plotted the dependences of the dimensionless temperature Θ_s in the direction of the flow of the fluid in the centre of the porous board for different axial thermal conductivities λ_z, i.e. for different orthotropies of thermal conducting $\gamma = \lambda_z/\lambda_r$. Assuming a low γ and a good heat transfer, the temperature of the fluid that passed comes much closer to the highest possible T_{s0} temperature than in the case of isotropic material. This is due to the fact that for small values of γ the temperature of a porous substance for $Z = Z_0$ is closer to the T_{s0} temperature. The effect of γ on the Θ_f isotherms for $0 < R < 1$ is shown in the informative picture in the Fig. 3.

CONCLUSIONS

The above-described method of calculation of temperature fields in porous substance as in fluid permits to determine the temperature curves for various geometrical and physical parameters. It is practicable to respect the anisotropy of porous materials as well as the cases where there are rather significant differences between the temperature of the fluid and that of the porous substance throughout the whole volume of the porous material. The whole situation can be characterized by three dimensionless parameters. From the calculated relations it follows that the highest efficiency is reached with a high radial value and a low axial value of thermal conductivity of the material of the porous board. The results of the method are used for an optimization of heat exchangers designed for throughflow-type helium cryostats.

REFERENCES

1. T. O. Niinikoski, Construction of sintered copper heat exchangers, *Cryogenics* **11**, 232–233 (1971).
2. R. B. Bird, W. E. Stewart and E. N. Lightfoot, *Transport Phenomena*. John Wiley, New York (1965).
3. L. Půst, Sintered heat exchangers for throughflow helium cryostats, Diploma Thesis, Faculty of Nuclear and Physical Engineering of the Czech Technical University, Prague (1976).
4. Jahnke-Emde, *Tables of Higher Functions*, Teubner, Leipzig (1952).